Journal of Applied Mechanics and Technical Physics, Vol. 40, No. 4, 1999

REGULARITIES OF THE STRETCHING AND PLASTIC FAILURE OF METAL SHAPED-CHARGE JETS

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UDC 532.52

The results of physicomathematical modeling obtained within the framework of continuum mechanics by numerical solution of the two-dimensional axisymmetric nonstationary problem of the dynamic deformation of a compressed elastoplastic bar of variable section are presented. Dependences of the quantitative characteristics of stretching and breakup of a shaped-charge jet (the coefficients of ultimate and inertial elongation and the number of individual elements formed in breakup) on the jet parameters and the jet material properties are revealed by calculations. The calculated dependences are compared with experimental data for plastically failing jets of copper and niobium, and the character of the dependences is explained from the physical viewpoint.

Introduction. Physicomathematical models for the deformation of a shaped-charge jet (SCJ) at the stage of uniform stretching preceding the necking process in the SCJ and breakup of the jet into separate nongradient elements are considered in [1, 2]. In these models, an SCJ element is represented as a cylindrical bar stretched at a constant value of the Lagrangian axial-velocity gradient. Because the indicated models are one-dimensional, they do not give an answer to the main questions: how and when does transition from the stage of uniform stretching (inertial stage) to the necking stage in a SCJ take place [1], how do separate nongradient elements and how do the geometrical and kinematic parameters of the SCJ elements and its material parameters influence the main quantitative characteristics of stretching and breakup of plastically failing SCJ?

The present work reports results of physicomathematical modeling of the process of stretching and plastic failure of SCJ in a more general formulation compared to the models of [1, 2]. The results are obtained within the framework of continuum mechanics by numerical solution of the two-dimensional axisymmetric nonstationary problem of the dynamic deformation of a compressible elastoplastic bar of variable cross section.

Apparently, this approach to examining the deformation of SCJ was first used by Chou et al. [3, 4]. Their studies were based on the Wilkins modification of the Lagrangian finite-difference method [5].

The main differences of the present work from the ones cited above are as follows. The deformation of SCJ elements was considered from the moment they formed by collapse of the corresponding elements of the shaped-charge liner to the moment of plastic failure with formation of separate nongradient elements. This required improving the Lagrangian finite-difference method [5] and extending it to the case of large strains. As a result of the calculations, the following quantitative characteristics of stretching and breakup of SCJ were determined: the coefficient of ultimate elongation n_{ult}, the coefficient of inertial elongation n_{i}, and the relative initial length a_0 of a jet segment that forms a separate nongradient element after plastic failure, which depends on the total number N of separate elements formed after SCJ breakup. Generalization of the numerical results reveals dependences of the quantitative characteristics of stretching and breakup on the SCJ parameters and material properties. The dependences are compared to the experimental data for plastically failing jets of copper and niobium, obtained by one of the authors in the middle of the 1970s. In

Bauman Moscow State Technical University, Moscow 107005. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 40, No. 4, pp. 25–35, July–August, 1999. Original article submitted October 30, 1996; revision submitted November 11, 1997.

0021-8944/99/4004-0571 $22.00

conclusion, the character of these dependences is explained from the physical point of view. The calculation and theoretical results reported in the present paper were obtained in the mid-1980s.

Main Content of the Physicomathematical Model. For the most part, the physicomathematical model proposed here has much in common with the model of [3, 4]. The problem is considered in the following formulation.

We study stretching of a chosen jet element bounded by planar sections with fixed Lagrangian axial coordinate z_0 in a coordinate system attached to one of the ends of this element. It is assumed that the jet material is a compressible, elastic, ideally plastic medium. The axisymmetric deformation of the SCJ element upon its stretching in cylindrical coordinates (r, θ, z) is described by a system of equations including differential equations of continuity, motion, and energy, kinematic relations, an equation of state that describes the compressibility of the material, the Prandtl–Reiss equations of plastic flow with the Mises plasticity condition, which describe the mechanical behavior of the material, the relations among the total strain tensors, spherical tensor, and stress deviator, and differential equations for the current Eulerian coordinates of individual points in the medium [5].

The initial conditions for the problem of the stretching of the chosen SCJ element (in what follows, the calculated element) are specified as follows. Since the length l_0 of the selected element is small compared to the overall length of the SCJ, as the initial distribution of the axial velocity, we specify the linear distribution $u_z = \dot{\varepsilon}_{z0} z_0$ determined by the local value of the initial axial-velocity gradient $\dot{\varepsilon}_{z0} = u_0/l_0$ (u_0 is the axial-velocity gradient between the planar sections bounding the calculation element). In contrast to [3, 4], the initial conditions for the radial velocity, stress-tensor components, stress deviator, and pressure are assumed, as in [2], to correspond to the initial axial-velocity gradient $\dot{\varepsilon}_{z0}$ according to the kinematic and dynamic relations of [1]. As additional initial conditions, we specify the initial perturbation $r_j = r_j(z)$ of the shape of the lateral surface with respect to a cylindrical surface with radius R_0.

The boundary conditions in our model of a stretched SCJ element are specified from the necessity of modeling the conditions of deformation of the element as part of the jet (the symmetry conditions at the ends of the element) taking into account the presence of a stress-free lateral surface and a symmetry axis [3–5].

Numerical Solution of the Two-Dimensional Nonstationary Problem of the Deformation of a High-Gradient Bar of Variable Cross Section with Large-Strains until Breakup. The features of numerical solution of the problem of the dynamic deformation of a high-gradient bar of variable cross section until the moment of its plastic failure are determined by the large strains experienced by the jet elements at the moment of breakup [1]. Under such conditions, the Lagrangian finite-difference method [5] cannot be used directly, as in [3, 4], in numerical calculations of the process with simultaneous solution of the problem of determining the characteristics of stretching and failure of the SCJ because of significant distortions of the difference grid and deterioration of the approximation of the differential equations by difference equations (for the typical case, the ratio of the axial and radial dimensions of the Lagrangian grid changes by two orders of magnitude during calculations). In this connection, in the numerical modeling reported in the present paper, the finite-difference method of [5] was employed in combination with an algorithm for reconstructing the grid. This permits one to continuously approximate the initial differential problem by a difference problem and, thereby, extend the possibilities of the Lagrangian approach to the case of flows with large strains. The algorithm of reconstructing the difference grid consists of periodic restoration of the grid regularity, sometimes, by increasing the number of grid nodes. Thus, the principle of reconstruction of [6] relating to the invariance of the positions of the grid nodes at the boundary of the calculation region was satisfied. In this case, this restriction was met for the free lateral area of the calculation element. The algorithm of reconstruction of the difference grid was tested by numerous calculations.

Substantiation of the Approach to Calculating the Quantitative Characteristics of SCJ Stretching and Breakup In [3, 4], the features of stretching of a SCJ were studied by analysis of development of initial perturbations of the shape of the lateral surface $r_j = r_j(z)$ of a SCJ element with respect to a cylindrical shape. This approach provide an adequate qualitative description of the main physical features of SCJ stretching.

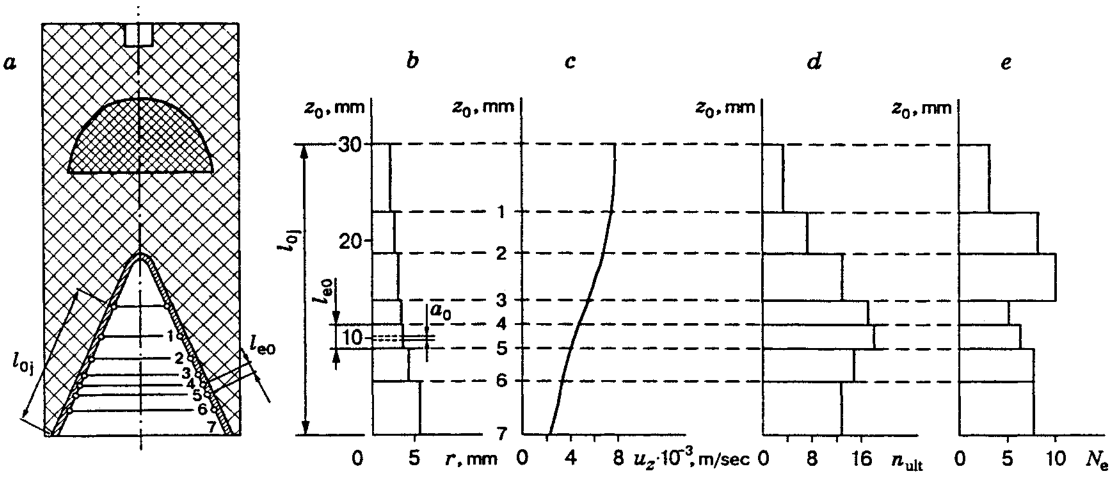

Fig. 1

Thus, the calculation in [3, 4] of the development of the initial harmonic surface perturbations

$$r_j = R_0 + A_0 \cos(2\pi z_0/l_0) \tag{1}$$

during stretching of a SCJ element treated as an elastoplastic bar predicts development of instability in the deformation process and formation of a neck in the weak planar section with axial Lagrangian coordinate $z_0 = l_0/2$. Necking is accompanied by redistribution of the axial and radial velocities over the length of the SCJ element, deformation localization in the necking region, and formation of nongradient regions with corresponding cessation of the motion in the radial direction. At the moment the neck reaches zero radius, the necking process culminates in separation of the element considered into two nongradient parts moving at different velocities [7]. This pattern of deformation and failure is in good agreement with experimental data for plastically failing SCJ [1], which allows one to use the method of [3, 4] in numerical modeling of SCJ stretching and in determining its quantitative characteristics as functions of the jet parameters and physicomechanical properties of the jet material. From [3, 4] it also follows that the cause of necking in stretching of SCJ elements are the internal stress in the jet, related to the elastoplastic properties of the jet material. Therefore, the instability developed in a SCJ that deforms in a natural fashion in the stage of formation of necks can be apparently regarded as plastic instability.

In the numerical modeling described here, we used experimental data for copper jets generated by laboratory 50-millimeter shaped charges. Figure 1a shows a diagram of such a charge of sensitized RDX with a lens of a TNT-based material and a shaped-charge liner of constant thickness with an opening angle of 50°.

The kinematic parameters of the SCJ produced by this charge were determined by the method of "labeled jet" proposed by V. M. Titov in 1956. Numerals 1–6 in Fig. 1a designate the positions of labels from a tungsten powder applied on the inner surface of the shaped-charge liner by means of shellac-rosin varnish, and numeral 7 corresponds to liner base, from which the tail element of the jet is formed. The axial velocities of motion of the tungsten-labeled SCJ elements formed from the corresponding elements 1–7 of the shaped-charge liner were determined by pulsed x-ray recording. In each experiment, not more than two labels were used, which were spaced rather far apart to eliminate their mutual influence. Figure 1b shows the initial SCJ configuration in the coordinates r–z_0, where the initial length l_{e0} of the labeled jet elements corresponds to the distance along the generatrix between the labels (see Fig. 1a) and l_{0j} is the initial length of the SCJ, equal to the length of the generatrix of the shaped-charge liner. Figure 1c shows the axial-velocity distribution u_z for the SCJ elements determined in experiments. The values of the coefficient of ultimate elongation n_{ult} are given in Fig. 1d and the number of separate nongradient elements N_e into which the corresponding SCJ segments were divided after plastic failure are given in Fig. 1e. The values of $n_{ult} = l_e/l_{e0}$ and N_e were determined by comparing x-ray photographs of a completely broken, unlabelled SCJ with a time–space z–t diagram of motion of the labeled jet elements. This made it possible to "refer" the broken jet segments, based on their overall length l_e and number of separate elements N_e, to particular elements of the initial jet configuration and their initial length l_{e0}. For example, the SCJ element formed by the liner segment located between labels 4 and 5 (in what follows, element 4–5) has initial radius $R_0 = 3.5$ mm, length $l_{e0} = 2.4$ mm, and axial-velocity gradient $\dot{\varepsilon}_{z0} = (u_{z4} - u_{z5})/l_{e0} = 3.18 \cdot 10^5$ sec^{-1}, and at coefficient of ultimate elongation $n_{\text{ult}} = 17.8$, it plastically fails to form $N_e = 6$ separate nongradient of elements. Assuming that separate elements have the same length, we can determine the initial length a_0 of a segment SCJ that forms a separate element after plastic failure: $a_0 = l_{e0}/N_e$. Vice versa, from the known length a_0, the number of separate elements formed by breakup of a SCJ segment with initial length l_{e0} is determined as $N_e = l_{e0}/a_0$. Generally (the quantity a_0 is variable), the number of separate elements formed upon breakup of the entire jet is defined by the integral expression $N = \int\limits_0^{l_{0j}} \frac{dz_0}{a_0}$ or its discrete analog

$$N = \sum_1^k l_{e0i}/a_{0i}, \tag{2}$$

where k is the number of SCJ segments for which the quantity a_0 can be considered constant ($k = 7$ for the experimental results in Fig 1).

According to the above principle of experimental determination of the quantitative characteristics of SCJ breakup, in numerical modeling they were determined as follows.

The number of separate nongradient elements formed upon breakup of the jet or jet segments was determined, according to (2), using the calculated dependence of the initial length a_0 of the SCJ segment that forms a separate element after plastic failure on the jet parameters. The coefficient of ultimate elongation $n_{\text{ult}} = a_{\text{fin}}/a_0$ (a_{fin} is the final length of this element after plastic failure). The coefficient of inertial elongation was calculated from the time t_i plastic instability begins to develop and deformation is localized in the region of necks. Because from the beginning of stretching of the SCJ element to this time, deformation of the element proceeds uniformly with retention of a nearly cylindrical shape, the coefficient of inertial elongation can be obtained from the kinematic dependence $n_i = 1 + \dot{\varepsilon}_{z0} t_i$, which is valid for gradient stretching of a cylindrical bar [1].

In calculating the quantitative characteristics of stretching and failure of a SCJ, it is of significance to choose the initial surface perturbation $r_j = r_j(z)$, which afterward initiates development of plastic instability. In experiments it is apparently impossible to obtain the surface perturbations parameters typical of real SCJ at the moment jet elements form. Theoretical approaches to estimating these parameters are also lacking. In this connection, the main requirement to the specified initial surface perturbation is the nondependence or weak dependence of the characteristics n_i, n_{ult}, and a_0 (or N_e and N) on the parameters of this perturbation. For the harmonic surface perturbation (1), this requirement is not met since in this case a calculation jet element of length l_0 is selected beforehand and it is assumed to form just one separate element after plastic failure i.e., one of the failure characteristics is predetermined: $a_0 = l_0$. It is necessary to use an initial perturbation whose parameters have minimum influence on the calculated values of the quantitative characteristics of SCJ stretching.

Such an initial surface perturbation was specified as the sum of harmonic perturbations with the same amplitude A_{i0} and different wavelength l_0/i:

$$r_j = R_0 + \sum_{i=1}^{8} A_{i0} \cos(2\pi z i/l_0), \tag{3}$$

and the minimum wavelength $l_0/8$ did not exceed the experimental initial length a_0. Maximal departure of the element shape from a cylindrical shape with radius R_0 takes place at the ends and determines the maximum perturbation amplitude $A_0 = 8A_{i0}$. For a difference grid with initial number of nodes on the axis $m_z = 17$ (which afterward increases according to the algorithm of reconstructing of the grid) the departure of the shape for the middle part of the calculation element does not exceed A_{i0}.

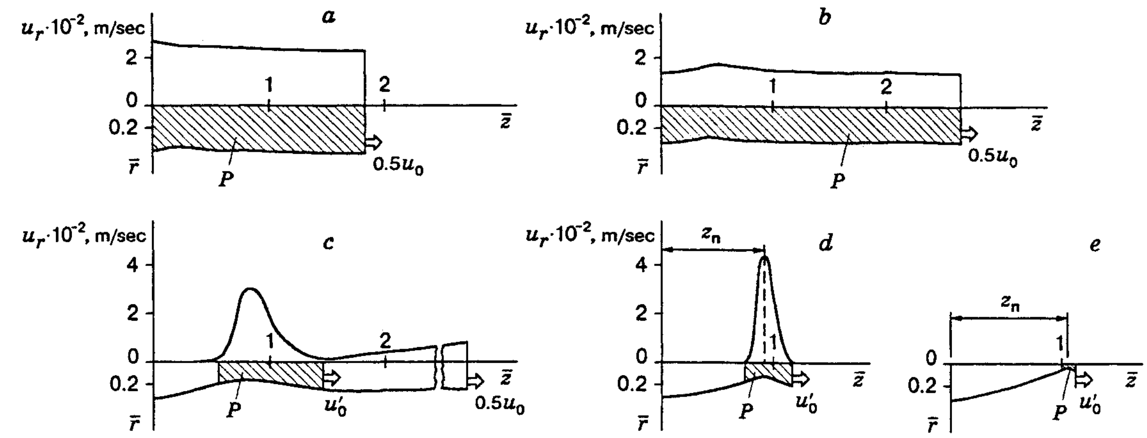

Fig. 2

During the numerical calculation of the stretching of a SCJ element with the initial perturbation (3), a numerical Fourier analysis of the shape of the lateral surface of the calculated SCJ element was performed. The coefficient of inertial elongation was determined from the time $t_{\rm i}$ the uniform deformation of the SCJ element ceased and the most rapidly growing perturbations began to develop.

Figure 2a–e shows the configurations of part of the calculation element $0 \leqslant z_0 \leqslant l_0/2$ and the axial distributions of the radial velocity u_r of the element's lateral surface ($\bar{r} = r/R_0$, $\bar{z} = z/R_0$) at times $\bar{t} = \dot{\varepsilon}_{z0}t = 6.4$, 9.6, 16.6, 17.7, and 20.0. Until the time $\bar{t}_{\rm i} = 9.6$, the velocity distribution has a uniform character (Fig 2a). At $\bar{t} \geqslant \bar{t}_{\rm i}$, plastic instability begins to develop and deformation localization is manifested with gradual formation of a separate element at the end $z_0 = 0$ (Fig 2b). On both sides of the developed neck, nongradient regions form in which u_r is close to zero. Since the nongradient segments formed upon deformation localization move at temporally constant axial velocity u_z [7] and their plastic deformation ceases, these segments were not considered in the calculation of the further necking. On the boundaries of these elements, we imposed new boundary conditions on the velocities $u_r = 0$ and $u_z = u_0'$, where u_0' is the calculated axial velocity of particles of the element on the nongradient segments formed (Fig 2c). As the dimensions of the nongradient segments increase, the calculated region P (dashed) was constantly diminished (Fig 2d), and the nodes of the difference grid were concentrated in the necking region. These made it possible to perform calculation until formation of a neck with a very small radius and thus model plastic failure of the SCJ (Fig 2e). From the axial Lagrangian coordinate of the neck $z_{0\rm n} = a_0/2$ and its Euler coordinate $z_{\rm n} = a_{\rm fin}/2$ at the moment of plastic failure, we determined the relative initial length $\bar{a}_0 = a_0/R_0$ of the SCJ segment that formed a separate element and the coefficient of ultimate elongation $n_{\rm ult} = a_{\rm fin}/a_0$.

As the calculations showed, use of the surface perturbation (3), which initiates development of plastic instability, as one of the initial conditions does not predetermine the characteristics of stretching and breakup of the SCJ. Thus, with variation in the geometrical and kinematic parameters of a SCJ element and the physicomechanical properties of the material over a wide range for the same initial surface perturbation (relative maximum deviation from the cylindrical shape $\bar{A}_0 = A_0/R_0 = 0.05$ and relative length of the calculation element $\bar{l}_0 = l_0/R_0 = 0.50$–$0.67$), the calculated stretching and breakup characteristics vary in the following ranges: $n_{\rm ult} = 7.3$–37.4, $n_{\rm i} = 4.0$–12.2, and $\bar{a}_0 = 0.06$–0.42. In contrast, variation in the perturbation parameters $\bar{A}_0$ and $\bar{l}_0$ over a wide range has a rather weak influence on the quantitative breakup characteristics of the same (in the physical sense) SCJ element (invariability of the initial radius, axial-velocity gradient, material characteristics, etc.). For example, when the length of the calculation element $\bar{l}_0$ changes by a factor of 2, the change in $n_{\rm ult}$ is not more than 10% and the change in $\bar{a}_0$ is less than 20%. Variation in the perturbation amplitude in the range $\bar{A}_0 = 0.025$–0.075 corresponds to a less than 10% change in the breakup characteristics.

To clarify this relative invariance of the characteristics of stretching and failure of SCJ determined using the initial surface perturbation (3), we analyzed the evolution of harmonic surface perturbations of the form (1). Analysis of the development of the initial perturbation (3) showed that by the beginning of the necking stage, jet segments with relative initial length $\bar{a}_0$ correspond to jet segments with relative length $a_i/R_i = (a_0/R_0)n_i^{1.5}$ [1], which is practically equal to the wavelength of a harmonic surface perturbation that grows most rapidly at this stage, where R_i is the radius of the SCJ element at the beginning of the necking stage. The calculated values of the relative wavelengths of harmonic perturbations that grow most rapidly at this stage turned out to be close to the value of 3 obtained in [3, 4]. For such perturbations, the coefficients of ultimate elongation are minimal compared to those of the other wavelengths, i.e., development of these perturbations on the jet is "favorable," primarily from the energy point of view since minimum energy dissipation is ensured in plastic deformation of the SCJ elements. The causes of the weak influence of the initial surface perturbation amplitude on the breakup characteristics can be explained from the results of another series of calculations of the evolution of the surface harmonic perturbations (1). It is found that in the inertial stage of stretching of a SCJ element, harmonic perturbations are not developed but are even partially suppressed. Furthermore, perturbations with larger initial amplitude are more heavily suppressed. As a result, by the beginning of the necking stage there is some smoothing of the relative amplitudes.

Influence of the SCJ Parameters and Physicomechanical Material Properties on the Quantitative Characteristics of SCJ Stretching and Breakup. Calculations of the quantitative characteristics of stretching and breakup of the SCJ showed that the determining parameters of this process are the geometrical parameter — the initial radius of a SCJ element R_0, the kinematic parameter — the initial gradient of the axial velocity $\dot{\varepsilon}_{z0}$, and the physicomechanical properties of the SCJ material — the density ρ_0 and yield point Y_0, whereas the characteristics of elasticity (shear modulus G) and compressibility (bulk modulus K) of the SCJ material can be ignored. The latter implies that the dependence of the SCJ breakup characteristics differs in character from the dependence of the parameters of the oscillation process that proceeds in the SCJ in the stage of its uniform deformation and depends largely on the bulk modulus of the SCJ material [2]. This difference can be explained by the fundamental difference between the process of radial oscillations at the early stage of stretching of the SCJ and the process of development of plastic instability at the necking. The first process is caused by the compressibility and inertia of the material, whereas the development of plastic instability is a purely deformation process, which occurs at the level of internal stresses of the order of the yield point and with a weak manifestation of the compressibility property.

With allowance for this and dimension theory, the values of the stretching and breakup characteristics of the SCJ must be determined by the value of one dimensionless complex, for example, the complex $\bar{U} = Y_0/(\rho_0\dot{\varepsilon}_{z0}^2R_0^2)$, which describes the relation between the plastic and inertial forces [2]. It should be noted that previously, Haugstad [8] and Yu. I. Fadeenko and L. A. Merzhievskii in the late 1960 suggested that this complex be used as the one determining the breakup characteristic of the SCJ.

To confirm the decisive role of the dimensionless complex $\bar{U}$ and establish the character and form of the dependences $n_{\rm ult} = f_1(\bar{U})$, $n_i = f_2(\bar{U})$, and $\bar{a}_0 = f_3(\bar{U})$, we performed numerical calculations with variation in all its four components according to the approach to determining the stretching and breakup characteristics of the SCJ described above.

Table 1 gives the initial data and numerical results of calculations for 21 versions of values of R_0, $\dot{\varepsilon}_{z0}$, ρ_0, and Y_0. The numerals in the second graph indicate to which labeled element of the SCJ from a 50-millimeter laboratory charge in Fig. 1 correspond the initial values of R_0 and $\dot{\varepsilon}_{z0}$.

Statistical processing of the calculations results presented in Table 1 confirmed the existence of a practically rigid functional dependence between the stretching and breakup characteristics of the SCJ and the dimensionless complex $\bar{U}$. For correlation coefficients not smaller than 0.98, the calculation results in logarithmic coordinates are approximated by linear functions corresponding to the power dependences

$$n_i = 2.78(\rho_0\dot{\varepsilon}_{z0}^2R_0^2/Y_0)^{0.32}; \qquad (4)$$

$$n_{\rm ult} = 5.38(\rho_0\dot{\varepsilon}_{z0}^2R_0^2/Y_0)^{0.39}; \qquad (5)$$

$$\bar{a}_0 = 0.65(Y_0/(\rho_0\dot{\varepsilon}_{z0}^2R_0^2))^{0.51}. \qquad (6)$$

In this case, relation (6) practically does not differ from the expression $\bar{a}_0 = a_0/R_0 = 0.65\sqrt{Y_0/(\rho_0\dot{\varepsilon}_{z0}^2R_0^2)}$,

TABLE 1

No.	Element	R_0, mm	$\dot{\varepsilon}_{z0} \cdot 10^{-5}$, sec^{-1}	ρ_0, kg/m^3	n $Y_0 \cdot 10^{-8}$, Pa	$\bar{U}^{-1}$	$\bar{a}_0$	$n_{\rm i}$	$n_{\rm ult}$
1	1–2	2.4	1.69	8900	2	7.32	0.261	5.13	11.2
2	1–2	2.4	1.69	8900	4	3.66	0.331	4.4	8.9
3	1–2	2.4	1.69	8900	6	2.44	0.422	4.06	7.35
4	2–3	2.8	2.56	8900	2	22.9	0.127	7.42	18.4
5	2–3	2.8	2.56	8900	4	11.4	0.189	6.14	14.3
6	2–3	2.8	2.56	8900	6	7.62	0.214	4.92	12.4
7	3–4	3.2	2.86	8900	2	37.3	0.105	9.63	21.1
8	3–4	3.2	2.86	8900	4	18.7	0.143	6.7	16.9
9	4–5	3.5	3.18	8900	1	110.3	0.060	12.2	37.4
10	4–5	3.5	3.18	8900	2	55.1	0.089	10.6	23.3
11	4–5	3.5	3.18	8900	3	36.8	0.106	9.4	20.4
12	4–5	3.5	3.18	8900	4	27.6	0.123	7.5	18.7
13	4–5	3.5	3.18	8900	5	22.1	0.125	7.4	18.4
14	4–5	3.5	3.18	8900	6	18.4	0.142	6.9	16.9
15	4–5	3.5	3.18	8900	10	11.0	0.184	6.25	14.4
16	4–5	3.5	3.18	2700	2	16.7	0.152	7.36	16.4
17	–	3.5	1.59	8900	2	13.8	0.185	5.94	14.6
18	4–5	1.75	3.18	8900	0.5	55.1	0.087	10.6	23.3
19	5–6	4.1	1.86	8900	2	25.9	0.118	8.19	18.9
20	5–6	4.1	1.86	8900	4	13.0	0.189	6.18	14.0
21	5–6	4.1	1.86	8900	6	8.63	0.222	5.32	12.3

from which it follows that the initial length of a separate element a_0 does not depend on the initial radius of the jet and is determined by a certain characteristic difference in velocity between the planar sections bounding it, $\Delta u_z = \Delta u_{\rm cr} = \dot{\varepsilon}_{z0} a_0 = 0.65\sqrt{Y_0/\rho_0}$, which depends on the strength and density of the SCJ material. In view of this, using relation (2), we can find the number of separate elements formed in plastic failure of the SCJ portion bounded by planar sections with Lagrangian axial coordinates z_{01} and z_{02} moving at axial velocities u_{z1} and u_{z2}:

$$N_{12} = \int\limits_{z_{01}}^{z_{02}} \frac{dz_0}{a_0} = \int\limits_{z_{01}}^{z_{02}} \frac{dz_0\, \dot{\varepsilon}_{z0}}{\Delta u_{\rm cr}} = \frac{u_{z2} - u_{z1}}{\Delta u_{\rm cr}}. \tag{7}$$

Assuming that u_{z1} is the velocity of the head element of the SCJ and u_{z2} is the velocity of the jet "tail," from (7) we obtain a formula for the total number N of separate elements formed upon failure of the entire SCJ. It is easy to see that relations (6) and (7), obtained by approximation of the numerical results, fit the so-called concept of the critical mass rate [9, 10].

Comparison of the Calculation Results with Experimental Data. The calculated dependences (5)–(7) (the coefficient of ultimate elongation $n_{\rm ult}$, the relative initial length $\bar{a}_0$ of separate SCJ elements, and their number on SCJ segments $N_{\rm e}$ or the total number N) were compared with experimental data on the breakup of plastically failing copper and niobium jets [11].

Approximation of experimental data (43 versions for copper SCJ generated by laboratory 50-millimeter shaped charges differing in the charge composition, design of the lens unit, and thickness and cone angle of the shaped-charge liner, and 9 versions for a niobium SCJ) by power relations of the form (5), (6) with a value of the yield point $Y_0 = Y_{01} = 10^8$ Pa gives good agreement between the exponent in the expression for the coefficient of ultimate elongation and its numerical value in formula (5). The exponent in the expression for the relative initial length of a separate element also nearly coincides with the calculated value [see (6)]. This implies that the calculation results do not contradict the experimental data on the form of the functional dependence of the SCJ breakup characteristics on its parameters.

Processing of the experimental data for the calculated dependences (5) and (6) by the least-square method gives values of the yield point Y_0 for the SCJ material for which there is best agreement between the coefficient of ultimate elongation (Y_{0n}) and the relative initial length of separate elements (Y_{0a}). For a

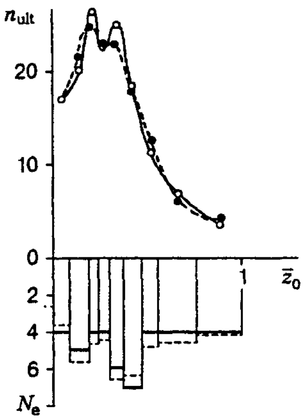

Fig. 3

copper SCJ, the yield points is $Y_{0n} = 0.46$ GPa and $Y_{0a} = 0.28$ GPa, and for a niobium SCJ, $Y_{0n} = 0.26$ GPa and $Y_{0a} = 0.32$ GPa. The quantity $Y_0 = 0.5(Y_{0n} + Y_{0a})$ can be adopted as the final estimate of the yield point. The calculated breakup characteristics of copper and niobium SCJ obtained for this value of the yield point are in good agreement with experimental data. Thus, the difference between the results for the ultimate elongation and the total number of separate elements formed after plastic failure of a copper SCJ exceeds 20% only in rare cases. The closest agreement is obtained for the results for a niobium SCJ, which are given in Fig 3 as distributions in the relative Lagrangian axial coordinate $\bar{z}_0 = z_0/l_{0j}$. The vertical lines show the locations of labeled SCJ segments (location of tungsten labels determining the division of the SCJ into segments with controlled initial and final length and with the number of the separate elements formed). The solid curves show experimental values and dashed curves show calculation results. In this case, the calculated and experimental distributions of the coefficients of ultimate elongation practically coincide. The total number of separate elements in the calculation $N = 47$ differs by approximately 10% from $N = 42$.

Thus, on the whole, the good agreement between the calculated and experimental results suggest that dependences (4)–(7) of the SCJ stretching and breakup characteristics on the jet parameters and physicomechanical material properties obtained by numerical physicomathematical modeling adequately describe the regularities of these processes for plastically failing SCJ.

Possible Physical Explanation of the Stretching and Breakup Characteristics of Plastically Failing SCJ. Dependences (4) and (6) for the coefficient of inertial elongation and relative initial length of separate elements obtained in the computations are close to the following:

$$n_{\rm i} \approx \sqrt[3]{\rho_0 \dot{\varepsilon}_{z0}^2 R_0^2 / Y_0}, \qquad \bar{a}_0 \approx \sqrt{Y_0/(\rho_0 \dot{\varepsilon}_{z0}^2 R_0^2)}. \tag{8}$$

The above substantiation of the results obtained, for example, on the value of $\bar{a}_0$, explains the causes of formation of separate elements of particular dimensions by their correspondence to surface perturbations that grow most rapidly at the necking stage or by manifestation of the so-called concept of the critical mass rate [9, 10]. One can attempt to consider the development of plastic instability of a SCJ from the viewpoint of the extremum principles [12] using the assumption of minimum energy dissipation in plastic deformation of the SCJ and simple energy relations for an incompressible rigidly plastic bar [1].

According to the relations from [1], the expression for the energy $E_{\rm d}$ dissipated in plastic deformation of a SCJ segment with mass M until its plastic breakup into separate elements of mass $m = \pi R_0^2 a_0 \rho_0$ can be written on the basis of the energy conservation law as

$$E_{\rm d} = \frac{M}{m}\Big(\frac{mY_0}{\rho_0} \ln n_{\rm i} + \frac{m\dot{\varepsilon}_{z0}^2 R_0^2}{16 n_{\rm i}^3} + \frac{m\dot{\varepsilon}_{z0}^2 a_0^2}{6} - \frac{m\dot{\varepsilon}_{z0}^2 a_0^2}{4} + A_{\rm d}\Big). \tag{9}$$

The first term in brackets characterizes the internal energy dissipated at the stage of uniform deformation of each separate SCJ element for $t \leqslant t_{\rm i}$. The second and third terms describe, respectively, the kinetic energy of the radial and axial motion of particles of a separate SCJ element at the beginning of the necking stage. The fourth term is equal to the kinetic energy of this element at the moment of breakup, when nongradient segments moving as a rigid whole form. The quantity $A_{\rm d}$ takes into account the work done by external (with respect to the separate element) forces at the necking stage. It cannot be determined analytically since numerical calculations using the two-dimensional model show that at this stage, internal stresses that arise on nongradient SCJ segments during their formation can be considerably smaller than the values typical of a cylindrical bar. We set this quantity equal to zero: $A_{\rm d} = 0$.

Using the condition $\partial E_{\rm d}/\partial n_{\rm i} = 0$, from relation (9) we determine the coefficient of inertial elongation, which does not depend on the mass of a separate element m or its relative initial length $\bar{a}_0$ and takes the form

$$n_{\rm i} = \sqrt[3]{\frac{3}{16}\,\frac{\rho_0 \dot{\varepsilon}_{z0}^2 R_0^2}{Y_0}},$$

which is qualitatively the same as the numerical result.

It can be assumed that minimum energy dissipation must also be ensured at the necking stage because of the definite relative initial length of separate elements $\bar{a}_0$. The expression for the energy dissipated at this stage can be written similarly to the expression for $E_{\rm d}$:

$$E_{\rm d.n.} = \frac{M}{m}\Big(\frac{m\dot{\varepsilon}_{z0}^2 R_0^2}{16 n_{\rm i}^3} + \frac{m\dot{\varepsilon}_{z0}^2 a_0^2}{6} - \frac{m\dot{\varepsilon}_{z0}^2 a_0^2}{4}\Big) = M\dot{\varepsilon}_{z0}^2 R_0^2\Big(\frac{1}{16 n_{\rm i}^3} - \frac{\bar{a}_0^2}{12}\Big).$$

The function $E_{\rm d.n.}(\bar{a}_0)$ is decreasing, and in the physical sense, it should be nonnegative. The minimum condition for this function $E_{\rm d.n.} = 0$ implies the relation $\bar{a}_0 = \sqrt{3/4}n_{\rm i}^{-3/2}$, which leads to the dependences $\bar{a}_0 = 2\sqrt{Y_0/(\rho_0 \dot{\varepsilon}_{z0}^2 R_0^2)}$, which is also in qualitative agreement with the calculation results (8).

Thus, the analysis performed suggests that the regularities of the process of stretching and breakup of plastically failing SCJ are determined by the fundamental principle of minimum energy dissipated during this process or the principle of minimum entropy production.

The dependences obtained in the present work (4)–(7) for the stretching and breakup characteristics of explosively produced metal shaped-charge jets can be used to calculate the kinematics of deformation of SCJ in free flight [11] and obtain information on the "structure" of plastically failing jets after breakup, including the number, dimensions, mass, and energy of the separate elements formed.

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